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Linear Optimization – Introduction

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European Structural and Investment Funds
Operational Programme Research, Development and Education

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Linear optimisation

Linear optimisation solves the problems of optimizing of some (linear) function (**objective function**) subject to some (linear) constraints. Several types of problems which are included in this part of Operations Research are production problem, diet problems, transportion problems and some others.

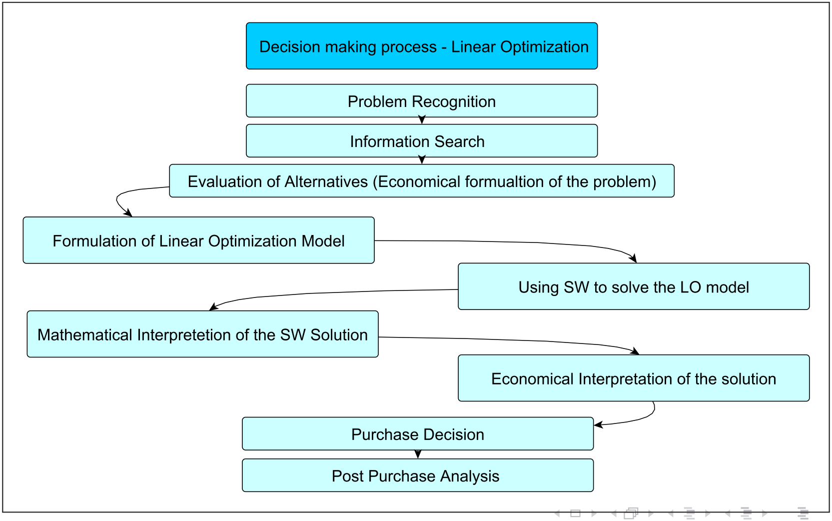
Protoexample

The Best Glass CO. produces high-quality glass windows. Now, they plan to use the remaining time of their production lines to start with the production of two new types of windows – let us call them Windows 1 and Window 2. All of these windows must go through three production lines, where the capacities of the lines are 60, 60, 85 hours. It is known that the unit of the first window type needs 2 hours at the first production line, 6 at the second one and 10 hours at the last production line. The unit of Windows 2 needs 10 hours at the first production line, 6 at the second one and 5 hours at the last production line.

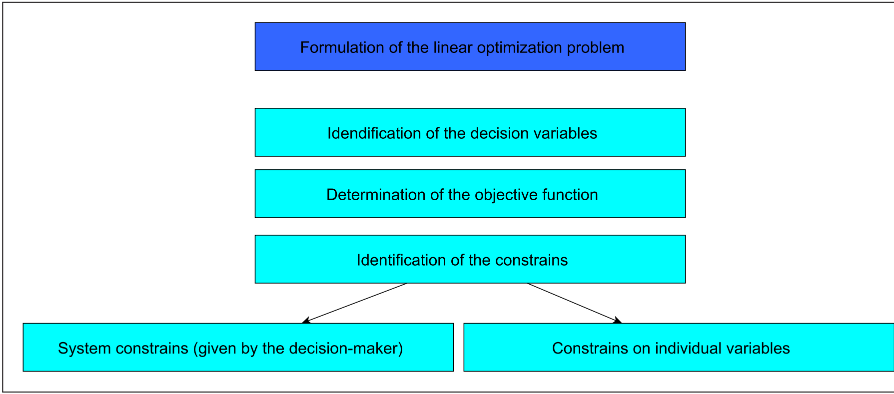
The marketing division considers that the company could sell as much of either product as could be produced and it is supposed that the profit from each unit of Windows 1 would be 30 thousand dollars and from each unit of Windows 2 45.

It is not clear which mix of these two products would be most

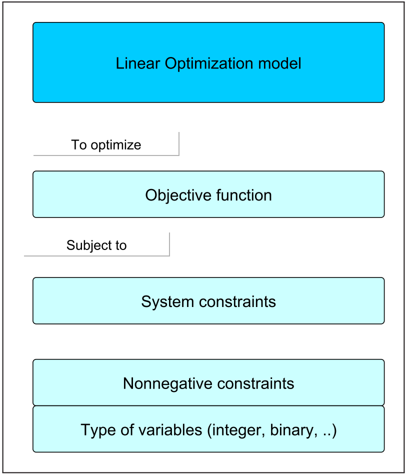
Solution Process



Steps of the Solution



Linear Optimization Model



Identification of the decision variables

In Prototype example, it can seem that there are two possibilities of decision variables – number of hours at each production line or number of produced units of each window type.
How to recognize which of these decision variables are the correct ones?

Decision variables

- x_1 the quantity of Windows 1 to produce;
- x_2 the quantity of Windows 2 to produce.

Then, if we solve the problem, we answer the question how many units of Windows 1 and how many units of Windows 2 the company should produce to optimize its profit. And this is what the manager would like to know.

Determination of the objective function

In Prototype example, the aim of the company is to maximize its profit. The profit depends on the number of produced units. It is known what is the profit from the each unit. Hence, the whole profit can by written as

$$30x_1 + 45x_2,$$

where x_1, x_2 are (as was written above) decision variables which gives us the number of units of Windows 1, resp. 2 to produce. Hence, we can write the objective function as

$$\max\ 30x_1 + 45x_2.$$

Identification of constraints

The Best Windows Comp. wants to maximize its profit, but they could not produce as many units as they want, they are limited by some restrictions – by the capacities of producing lines.
We know (for Line 1), that there is 60 hours available and that each unit of Windows 1 spends here 2 hours. Hence, if we produce x_1 units of Windows 1, we use $2x_1$ of the capacity fo the first production line. Each unit of Windows 2 needs 10 at this line, hence x_2 units needs $10x_2$ hours. Therefore, we can write:

$$2x_1 + 10x_2 \leq 60.$$

Similarly, we get for the second line:

$$6x_1 + 6x_1 \leq 60$$

and for the third line:

$$10x_1 + 5x_2 \leq 85.$$

Identification of constraints

We must add constraints which are not explicitly written in the problem but which must be fulfilled too. Typically, non-negativity of variables. In the prototype example, the decision variables are the number of produced units. Hence, it is clear, that we can not produce negative number of units, so we must add constraints on non-negativity of variables:

$$x_1, x_2 \geq 0.$$

Now, we have the whole linear optimisation model of our prototype example:

$$\begin{aligned} &\max 30x_1 + 45x_2 \\ \text{subject to } &2x_1 + 10x_2 \leq 60, \\ &6x_1 + 6x_2 \leq 60, \\ &10x_1 + 5x_2 \leq 85, \\ &x_1, x_2 \geq 0. \end{aligned}$$

Typical Types of Constrains 1

It is needed to produce the same amount of Windows 1 as Windows 2.

In fact, it is very easy condition, let us recall that x_1, resp. x_2 is an amount of produced Windows 1, resp. Windows 2. So, if the amounts should be the same, we can write it down as

$$x_1 = x_2.$$

Typical Types of Constrains 2

It is needed to produce at least as many units of Windows 1 as Windows 2.

The second type of the conditions is very similar. We have to produce more Windows 1 than Windows 2, hence we can write

$$x_1 \geq x_2.$$

Typical Types of Constrains 3

We have to produce at least 5 more Windows 1 than we produce Windows 2.

We know that we produce x_2 of Windows 2, hence we should add 5 more, it is $x_2 + 5$ and produce at least such amount of Windows 1 (amount of Windows 1 is denoted by x_1. Therefore, we obtain:

$$x_1 \geq x_2 + 5.$$

Typical Types of Constrains 3

How to check the solution?

Let us suppose that we produce 4 Windows 2. In such a case we should produce at least $4 + 5$ Windows 1, so x_1 should be equal to or bigger than 9. Now, let us use the example number (4) in our constraint:

$$x_1 \geq x_2 + 5 = 4 + 5 = 9,$$

hence we have

$$x_1 \geq 9,$$

what is right. So our constraint is correct.

Typical Types of Constrains 4

We need to produce at least twice as many Windows 1 as Windows 2.

It means if we produce x_2 Windows 2, than twice as many Windows 1 as Windows 2 is equal to $2x_2$, hence the constraint can be written as

$$x_1 \geq 2x_2.$$

If we are not sure if we are right, we can again choose some value for x_2 and check our solution as was shown in the previous case.

Typical Types of Constrains 5

Using of Percentages in the conditions

Let us suppose following condtion: at least 30 percent of the production is Windows 1. Therefore, we can immediately write that $x_1 \geq stg$, where *stg* is 30 percent of the production. (Suppose that percents are understand as percent of pieces.) What is the whole production? It is $x_1 + x_2$, than 30 percent of $(x_1 + x_2)$ is $0.3(x_1 + x_2)$. Hence, the constraint is in the form:

$$x_1 \geq 0.3(x_1 + x_2),$$

or

$$0.7x_1 - 0.3x_2 \geq 0.$$