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Linear Optimization – Graphical solution

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Linear optimization – graphical solution

Linear optimization solves the problems of optimizing of some (linear) function (**objective function** subject to some (linear) constrains. Several types of problems which are included in this part of Operation Research are production problem, diet problems, transport problems and some others.

If the problem contains only 2 variables, we can solve it in graphical way.

Protoexample

The Best Glass CO. produces high-quality glass windows. Now, they plan to use the remainding time of their production lines to start with the production of two new types of windows – let us call them Windows 1 and Window 2. All of these windows must go through three production lines, where the capacities of the lines are $60, 60, 85$ hours. It is known that the unit of the first window type needs 2 hours at the first production line, 6 at the second one and 10 hours at the last production line. The unit of Windows 2 needs 10 hours at the first production line, 6 at the second one and 5 hours at the last production line.

The marketing division considers that the company could sell as much of either product as could be produced and it is supposed that the profit from each unit of Windows 1 would be 30 thousand dollars and from each unit of Windows 2 45.

It is not clear which mix of these two products would be most

Graphical solution

Step 1

Display the set of **feasible solutions** – it is an intersection of all constrains.

Step 2

Identify the **optimum solution** by using of objective function.

Mathematical model of Prototype example

First, let us recall the mathematical model of our Prototype example:

$$
\begin{aligned}
\max\ 30x_1 + 45x_2 & & \\
\text{subject to } 2x_1 + 10x_2 &\leq 60, \\
6x_1 + 6x_2 &\leq 60, \\
10x_1 + 5x_2 &\leq 85, \\
x_1,\ x_2 &\geq 0.
\end{aligned}
$$

How to display the constrains?

Let us start with the displaying of the first constrain:

$$2x_1 + 10x_2 \leq 60.$$

This inequality can be displayed as a half-space which is bounded by the line given by the formula

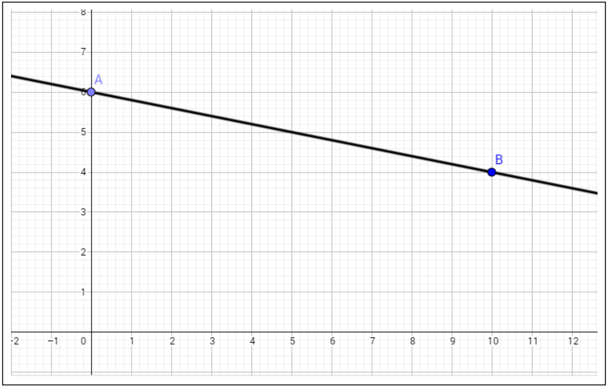
$$2x_1 + 10x_2 = 60.$$

Each line is given by two points, hence, let us find to point of this line. First, let us suppose that $x_1 = 0$. If we fit it into the equation we get

$$2 \cdot 0 + 10x_2 = 10x_2 = 60,$$

so we obtain $x_2 = 6$, therefore, the first point of the line is $[0, 6]$. Similarly, we get the second point of the line $[10, 4]$. So, we have the boundary of the half-space.

How to display the constrains?



Which half-space is the correct one?

Which part of the space is such that for all points at this set satisfy the inequality

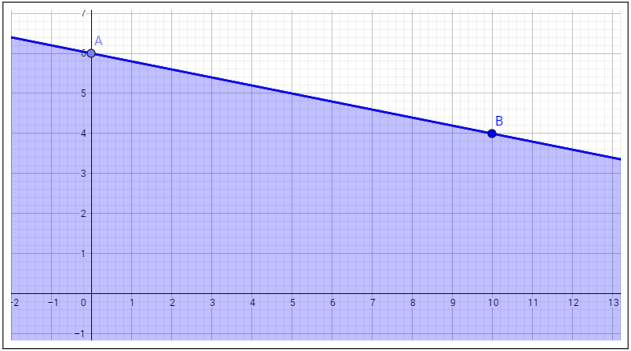
$$2x_1 + 10x_2 \leq 60?$$

Let us choose some point which does not lie on the boundary, for example point $[0, 0]$ and let us check if the point satisfies the inequality:

$$2 \cdot 0 + 10 \cdot 0 = 0 \leq 60.$$

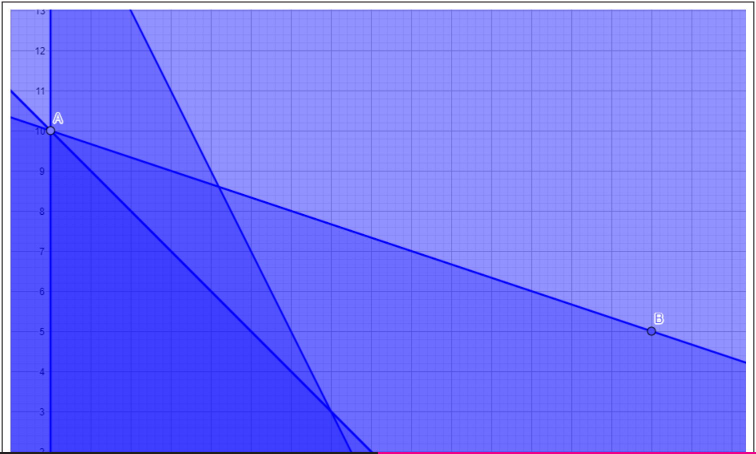
The inequality is fulfilled, so the searched half-space is such that which includes the point $[0, 0]$, see the following picture.

Displaying of the first constrain



Construction of the set of feasible solutions

In similarly way, we can construct the other conditions and the intersection of all these condition gives us the set of feasible solutions.



Types of feasible sets

Non-empty bounded set of feasible solutions

This case is the typical one – there exists non-empty bounded set of feasible solutions. In such a case there exists at least one optimum solution.

Empty set of feasible solutions

If the intersection of half-spaces of conditions is empty, then there is no feasible solution and do not exist a optimum one.

Unbounded set of feasible solutions

Sometime (typically when the setting of the example is not correct) the set of feasible condition is unbounded and the objective can grow up to infinity, in such a case the do not exist optimum solution, the objective function is unbounded.