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Multiple Criteria Decision Making (MCDM) – Introduction

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Multiple criteria decision making (MCDM)

MCDM is a part of multi-attribute decision making, where we search for the best solution in case when we take into account several objectives under some conditions. Especially, the MCDM is a case, when we have a (finite) list of possible alternatives and we know conditions under which we want to make a decision and also know objectives.

Typically, in MCDM there do not exist a optimum alternative. Usually, we have a list of possible "good" alternatives and our aim is to determine the "best" one, in fact the **compromise** one. (Because the choice of the best one is subjective.)

Decision matrix

MCDM – we have a list of all possible alternatives evaluated under several criteria. Hence, we can write down a **decision matrix** $\mathbb{R}$, where in rows we have alternatives and in columns are considered criteria. Therefore, the element of the matrix r_{ij} gives us the evaluation of the i-th alternatives according to the j-th criterion.

Prototype example

We want to buy a tent. We are interesting in the weight of the tent, waterproof rating, expert evaluation and price. We are thinking about following five types of tents (we like them, they have such properties which we need), the data are in the table.

Produkt	**weight**	**waterproof**	**expert**	**price**
Type 1	2.4 kg	1200mm	3	3990 CzK
Type 2	2.5 kg	1600mm	2	4500 CzK
Type 3	2.7 kg	1500mm	2	4700 CzK
Type 4	3.5 kg	400mm	5	1990 CzK
Type 5	3 kg	1000mm	4	2500 CzK

Decision matrix

Using the data from the previous slide, we can construct a decision matrix as follows.

$$\mathbb{R} = \begin{pmatrix} 2.4 & 1200 & 3 & 3990 \\ 2.5 & 1600 & 2 & 4500 \\ 2.7 & 1500 & 2 & 4700 \\ 3.5 & 400 & 5 & 1990 \\ 3 & 1000 & 4 & 2500 \end{pmatrix} \tag{1}$$

The decision matrix has five rows (= number of alternatives) and four columns (= the number of criteria). The element r_{ij} gives us the value of alternative i under criterion j.

Criteria types

As we can see in our Prototype example, we can have typically two criteria types – cost type (in our example weight and price) and profit type (in our example waterproof rating and expert evaluation).
It is important to keep the type of criteria during our analysis. Some methods need to have all criteria of profit type, then it is necessary to transform cost type criteria into profit type (methods how to do it will be discussed later).

Existence of feasible solution

Similarly to linear optimization, first question is, if the feasible solution of this problem exists. The **feasible solution** is any solution which satisfy all conditions given by decision maker. Typically, we have in our first table only feasible solutions (typically, we do not include alternatives which do not match with our conditions). Sometimes, we have a list of several possible alternatives and then we state some conditions and choose the feasible alternatives, then the set of feasible solution can be narrower then the previous list of possible alternatives. (For example, I am in the shop, where are 20 tents, and I can do the list of all of them. But, in fact, all of them are not acceptable for me - for example some of them are too expensive, some of them do not have enough good waterproof rating, so I have to narrow the list of tents to feasible ones.)

Existence of compromise solution

If the feasible solution exists, then the compromise exists, too.

Feasible and compromise solution in Prototype example

If the decision maker is willing to buy any of the five tents, then all five alternatives are feasible. To determine the compromise solution, we need first to develop some of the MCDM methods.

Does there exist unique optimal solution?

Typically, not.

It is the main difference between MCDM and most other methods of operation research, typically there is not only one optimal solution. It is a reason, why we speak about **compromise solution**. The solution of these methods depends on the preferences of decision maker, on the choice of method, on the choice of normalization of the data, on the type data transformation and so on.

Special case.

The only case, when there is unique optimal solution is the case when one of the alternatives dominates all other ones.

Nondominated alternatives, Pareto optimal alternatives

Dominated alternative

We say that the alternative A is **dominated by the other one** if there exists the other alternative which is under all criteria better than or same as the alternative A and at least at one criterion it is better.

It is clear from the definition, that the dominated alternatives cannot be the compromise one (there exists some other alternative which is better).

Nondominated alternative, Pareto optimal one

We say that the alternative is nondomited (or Pareto optimal) if it is not dominated by any other alternative.

Basic properties of MCDM methods

As we mentioned above, typically there is no unique solution of MCDM problem, the solution depends on the choice of weights, the choice of method, the choice of dat standardization. However, to recognize a "good" method of MCDM? The solution of these methods should satisfy the following conditions.

Pareto optimal solution

The solution is Pareto optimal

The solution given by the method must be Pareto optimal (nondominated) alternative. We say that the alternative A is **dominated** by the alternative B, if the alternative B is under all criteria better than (or the same as) alternative A and at least under one criterion it is better than alternative B. We also say that alternative B **dominates** alternative A.

The alternative A is **nondominated or Pareto optimal** if there is no alternative which it dominates.

Determination, Uniqueness

Determination

We want such method that for any set of feasible solution it gives us a compromise solution.

Uniqueness

The method should give us the unique solution (after the setting of weights.

Invariance to the ordering

Invariance to the ordering of criteria and alternatives

The choice of the compromise alternative should not depend on the ordering of criteria or alternatives.

Remark

It is very trivial condition which says that for example the result of the tender cannot depend on the order of candidates - if we sort the candidates by their names or by the names of their companies or by the data of receiving their application or by their offer - the result should be still the same.

Invariance to measure units

Invariance to measure units.

The choice of compromise alternative should not be affected by the choice of measure units in which we evaluate the criteria. For example, if we want to choose the best tent according to the weight, the result should be the same if we set the weight in kilograms, libras or grams.

Invariance to addition of nonoptimal alternatives

Invariance to addition of nonoptimal alternatives.

We should choose the same alternative does not matter if we added some non optimal (for example) dominated alternative to the list of feasible alternatives. The choice should be also the same if we remove from the list of alternatives all dominated alternatives or not.

Fairness of the method

Fairness of the method.

The method should allowed to choose any of nondominated alternatives by the setting of appropriate weights.

Properties of the MCDM methods

However, the above mentioned properties of MCDM methods seem to be rational, we will see in the following lectures that some of them are not (unfortunately) satisfied by all methods.