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Multiple Criteria Decision Making (MCDM) – Part 2

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Multiple Criteria decision making (MCDM) - summary of the previous lecture

Our aim in MCDM problems is to choose the compromise alternative from the list of alternatives which are evaluated under several criteria.

However, the choice of compromise solution depends on the choice of the weights and MCDM method, there are some basic properties which every method should satisfied and we never should choose a dominated alternative.

Prototype example (from the previous lecture)

We want to buy a tent. We are interesting in the weight of the tent, waterproof rating, expert evaluation and price. We are thinking about following five types of tents (we like them, they have such properties which we need), the data are in the table.

Produkt	**weight**	**waterproof**	**expert**	**price**
Type 1	2.4 kg	1200mm	3	3990 CzK
Type 2	2.5 kg	1600mm	2	4500 CzK
Type 3	2.7 kg	1500mm	2	4700 CzK
Type 4	3.5 kg	400mm	5	1990 CzK
Type 5	3 kg	1000mm	4	2500 CzK

Is any alternative dominated?

Is Type 1 dominated by Type 2?

First, let us compare the Type 1 with Type 2 under criterion weight – the Type 1 is better than Type 2, hence it cannot be dominated by Type 2.

Is Type 2 dominated by Type 1?

First, let us compare the Type 1 with Type 2 under criterion weight – the Type 1 is better than Type 2, it is OK. Let us continue with waterproof, under criterion waterproof the Type 2 is better than Type 1, hence it cannot be dominated by Type 1.

Is any alternative dominated?

Yes, Type 3 is dominated by Type 2.

Under **all** criteria Type 2 is at least such good as Type 3 and under some of them it is strongly better.

Type 3 is not Pareto optimal

It is dominated alternative, so it is not Pareto optimal, hence we can remove it from the analysis (however, it is not necessary; we know, that it could not be a compromise alternative).

Dominated alternatives – the conclusion

Non-dominated strategies

If we continue, we find out that there is only one dominated strategy, strategy number 3. All others are non-dominated. Therefore, only alternative number 3 can be removed from the decision making process.

Utopia and Nadir alternative

In many methods MCDM, we use so called **Utopia alternative** and **Nadir alternative**. Both of these alternatives are hypothetic alternatives, first one has the best possible values under all criteria, the second one has the worst evaluation under all criteria.

Utopia and Nadir alternatives in Prototype example

$$\text{utopia} = (2.4; 1600; 2; 1990),$$

$$\text{nadir} = (3.5; 400; 4700).$$

Methods of MCDM

As was mentioned, MCDM methods usually do not give unique solution. The solution depends on the choice of weight and also on the choice of the method. In this part we will go through the methods for the choice of weights and through MCDM methods.

Ordinal and cardinal information

Methods MCDM depend also on the information type – which information they need (about rank of alternatives under criteria or about the criteria prefences) and which information they give us.

Ordinal and cardinal information scale

Ordinal information scale

The ordinal information scale is the information scale, when we know only the rank. We can only rank the alternatives, criteria, we are not able to compare distances between them, we do not know what is the distance between alternative on the first and second place. We only know, what is the first, what is the second. What is better and what is worst but not how much.
In case of ordinal information scale we can not use any mathematical operations.

Cardinal information scale

In the case of the cardinal information scale, we can measure how much is the alternative better than the other one. We have more information about the rank than in ordinal information scale.
In such a case we can also apply mathematical operations.

Criteria weights in MCDM

What are weights?

Criteria weights give us the information about the importance of the criteria from the point of decision-maker's view. If the criterion is more important for the decision-maker, than the weight is bigger. Weights should be positive numbers (if the weight is equal to zero, it means that we do not take a care about this criterion, it has no importance for us). Typically, we ask to have **standardized** weights, it means the sum of weights should be equal to one.

Construction of criteria weights

The choice of criteria weights is subjective, weights give us the information about decision-maker criteria preferences. Hence the construction of weights must be done together with decision-maker. It is very important that the decision maker understand well to the weights role in decision making process, to its interpretation.
The different choice of weights usually means the different result of MCDM.
Usually, we do the choice of weights in two steps. First, we discuss with the decision-maker and we set the weights. Then, we explain him the meaning of weights and ask him if he agree with our setting. If not, we go back to the first step.

Basic method of weight construction

Equal weights method

In case, when we have no information about criteria preferences, the only way how to construct weights is to suppose equal importance of all criteria. Since we need to have the sum of weights to be equal to one and we want to have all weights to be equal, so in case of n criteria, we set

$$w_i = \frac{1}{n}, \quad \text{for all} i \in \{1, \dots, n\}.$$

Rank (sum) weight method

This criteria weight method is based on ordinal information about criteria ordering, therefore, it gives us also only ordinal information. How to apply this method? First, we assign to each criterion as many points as is the order of the criterion in the list of all criteria. More precisely, if we have n criteria, then the most important criterion has n points, the second one $n-1$ points and so on. The less important one has 1 point. Then, we standardize these points into the weights. We can write:

$$w_i = \frac{n+1-r_i}{n(n+1)/2},$$

where r_i is the order of the i-th criterion in importance, $i = \{1, \ldots, n\}$.

Reference point method

The construction technics of this method is similar to the previous one, however this method can be used only in case when we have cardinal information about decision maker criteria preferences, hence the weights give us also cardinal information about criteria preferences.

Since the cardinal criteria preference information is known, we can ask decision maker to assign points to each criterion which explain the importance of this criterion for him. If we receive such points, we get the weights by standardization again, hence we set

$$w_i = \frac{p_i}{\sum_{i=1}^{n} p_i},$$

where p_i are points assigned to i-th criterion.

Many modification of this method exists. One of the possible way is setting of the upper bound of points for one criterion (typically 10 or 100 points). The other possibility is to determine that it is to divide just 100 points among all criteria – is is called Point

Fuller triangle method

To use Fuller triangle method we need decision maker to be able to decide between each two criteria which of them is more important for him. Typically, it can be used in case, when we have a large number of criteria and it is diffucult for us to order them in one step.

Fuller triangle method – procedure

First, we construct Fuller triangle, it is a triangle which contains or possible pair of criteria. Then we compare each two criteria and we add one point to each criterion which is preferable to the other one. If we find a pair of criteria which are indiferent for the decion maker, we can add 1/2 point to both criteria.
In the end, we sum points for each criterion and we standardize the points in the same way as in reference point methods.

Fuller triangle method – modifications

Sometime, it can happen in this method that some of the criteria have zero points. Hence, after standardization, we would have the weight of this criterion equal to zero, what means no importance for the decision maker. However, we suppose, that the less important criterion from the DM point of view has a small importance for the DM but not equal to zero (what means no importance). Therefore, in this case we can before the standardization to add 1 point for each criteria.

Remark

In case, when the decision maker is able well-order the criteria preferences, the less preferable criterion has a weight zero and after modification Fuller triangle method gives the same weights as Rank sum weight method.

Saaty method

Fuller triangle method uses only ordinal information about preference between criteria. Hence, it is easy to run such method, however in case when we have cardinal information, we lost this part of information, what is a shame.

Construction

First, we need to construct so-called Saaty matrix $\mathbb{S}$, where we compare each pair of criteria by points from $1/9$ to 9. The s_{ij} should present w_i/w_j.