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Linear Optimisation in DEA

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Optimisation Problem

A general solution of DEA models uses linear optimisation. The basic idea is straightforward, for each unit we search for weights for inputs and outputs such that the efficiency of the unit is as height as is possible.
In mathematical formulation, we search for weights under which the efficiency of given unit is maximally and efficiencies of all units are less than or equal to one.

Basic Notation

x_{ik} – i-th input of k-th unit, $i = 1, 2, \ldots, m,\ k = 1, 2, \ldots, p,$

y_{jk} – j-th output of k-th unit, $j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, p.$

Weights which are assigned to each input, resp. output are denote by u_i, resp. v_j.

Expression of Efficiency

$$\text{efficiency} = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}},$$

in mathematical formulation:

$$e_k = \frac{\sum_{j=1}^{n} v_j y_{jk}}{\sum_{i=1}^{m} u_i x_{ik}}, \quad k = 1, 2, \ldots, p,$$

where u_i, v_j are weights for inputs and outputs and x_{ik} is the value of i-th iput of k-th unit and y_{jk} gives the value of j-th output of k-th unit.

Optimisation Model

$$e_H = \frac{\sum_{j=1}^{n} v_{jH} y_{jH}}{\sum_{i=1}^{m} u_{iH} x_{iH}} \quad \rightarrow \quad \max,$$

subject to:

$$\frac{\sum_{j=1}^{n} v_{jH} y_{jk}}{\sum_{i=1}^{m} u_{iH} x_{ik}} \leq 1, \ \forall k = 1, 2, \ldots, p,$$

$$v_{jH} \geq 0, \ \forall j = 1, 2, \ldots, n,$$

$$u_{iH} \geq 0, \ \forall i = 1, 2, \ldots, m.$$

CCR input oriented model

$$e_H = \sum_{j=1}^{n} v_{jH} y_{jH} \to \max$$

$$\sum_{i=1}^{m} u_{iH} x_{iH} = 1$$

$$-\sum_{i=1}^{m} u_{iH} x_{ik} + \sum_{j=1}^{n} v_{jH} y_{jk} \leq 0, \ \forall k = 1, 2, \ldots, p,$$

$$u_{jH} \geq 0, \ \forall j = 1, 2, \ldots, n,$$

$$u_{iH} \geq 0, \ \forall i = 1, 2, \ldots, m.$$

Prototype Example

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he district includes seven towns. The town offices are evaluated by wage costs (mill. CzK per year), operating costs (mill. CzK per year), value of municipality property; and by the number of inhabitants as is shown in the following table

Office	A	B	C	D	E	F	G
Wages	3,5	3	2,8	4	3,8	3,6	3,9
Op. Costs	1,5	1,4	1,6	1,7	1,3	1,25	1,8
Inhab.	1020	900	1200	1300	1100	800	1150
Property	50	52	48	55	53	46	54

Mathematical Model

$$
\begin{aligned}
e_1 = 1020u_{11} + 50u_{21} &\rightarrow \max \\
3,5v_{11} + 1,5v_{21} &= 1 \\
-3,5v_{11} - 1,5v_{21} + 1020u_{11} + 50u_{21} &\leq 0 \\
-3v_{11} - 1,4v_{21} + 900u_{11} + 52u_{21} &\leq 0 \\
-2,8v_{11} - 1,6v_{21} + 1200u_{11} + 48u_{21} &\leq 0 \\
-4v_{11} - 1,7v_{21} + 1300u_{11} + 55u_{21} &\leq 0 \\
-3,8v_{11} - 1,3v_{21} + 1100u_{11} + 53u_{21} &\leq 0 \\
-3,6v_{11} - 1,25v_{21} + 800u_{11} + 46u_{21} &\leq 0 \\
-3,9v_{11} - 1,8v_{21} + 1150u_{11} + 54u_{21} &\leq 0 \\
u_{j1} &\geq 0, \ j = 1,2, \\
v_{i1} &\geq 0, \ i = 1,2.
\end{aligned}
$$

Solution

Variable	Value
e_1	0,909864
v_{11}	0,135
v_{21}	0,3516
u_{11}	0,0003
u_{21}	0,012