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Optimisation in Project Management – Crashing

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Motivation

It is possible to finish the project early than the CPM result is? What are the cost in such a case?

Should we pay more (extra bonus for overtime, overwork, extra workers, special technology and so on) for their realization of activities to finish the project earlier and obtain a bonus?

Direct and Indirect Costs

Direct costs

They are attributed to a specific activity, e.g. labor, raw materials, and equipment rental costs. Direct cost grows up if we crash the activity (we need more people, special material, bonuses and so on).

Indirect costs

They cannot be directly attributed to a specific activity, they are connected with the whole project, e.g. management, general administration, rental and utility costs.
Indirect costs grow up with the length of the project, so if the whole project is crashed then the indirect cost is reduced.

Protoexample

ALEA comp.

ALEA comp. needs to finish a planned project within 12 months. The project has 4 activities – A, B, C, D. The project manager found out that it is not possible to finish the project in time. So, it would be necessary to crash any activities to be able to finish the project on time. The project manager wrote down also the shortest possible duration of each activity (in months) and estimate the cost for each activity in such a case. Which activities should be crashed to finish the project in time with minimal cost? It is known that indirect cost is $\$6,000$ per month. Is it better to finish the project in time or to pay a penalty $\$8000$?

Protoexample

Act.	Pred.	Normal time	Crash time	Normal cost.	Crash cost
A		8	5	$ 25,000	$ 40,000
B		9	7	$ 20,000	$ 30,000
C	A	6	4	$ 16,000	$ 24,000
D	B	7	4	$ 27,000	$ 45,000

Solution to ALEA comp.

When all activities take its normal durations the project will take 16 months. The critical activities are B and D. Total cost is $\$184,000$ (indirect cost $16 \cdot 6 = 96$ and direct cost $25 + 20 + 16 + 27 = 88$). If we want to finish the project in shorter time, we need to decide which activities we will crash and what will be the price for it (on the other hand we save on indirect cost).

Solution to ALEA comp.

When we want to solve such a problem, we first need to know which activities is possible to crash and with what is the cost per week saved. Hence, we prepare a table, where we denote, how can be the activity crashed and we estimate for each activity the cost per one week saved using the following formula:

$$\frac{\text{crash cost} - \text{normal cost}}{\text{normal duration} - \text{crach duration}}.$$

Solution to ALEA comp.

Let us do it for Prototype example, we put it into the following table.

Activity	Maximal reduction time (months)	cost per 1 month saved
A	3	$ 5,000
B	2	$ 5,000
C	2	$ 4,000
D	3	$ 6,000

The algorithm of the Solution

1. Use CPM with the normal duration of the activities to identify the critical path. Compute direct, indirect, and total costs in this case.
2. Choose the activity on the critical way (one activity at each critical paths in case of more than one critical paths) with the smallest cost per one week saved which is still possible to crash.
3. Crash the chosen activity(ies) and compute direct, indirect, and total costs in this case.
4. Continue with repeating steps 2 and 3 till it is possible to crash or till the total cost goes down or till we catch the duration of the project we need.

Tableau Solution

Total t.	cr. act.	Δt	Δc	in. c.	direct c.	total c.
16	-	-	-	$ 96,000	$ 88,000	$ 184,000
15	B	1	$ 5,000	$ 90,000	$ 93,000	$ 183,000
14	B	1	$ 5,000	$ 84,000	$ 98,000	$ 182,000
13	D	1	$ 6,000			
	C	1	$ 4,000	$ 78,000	$ 108,000	$ 186,000
12	D	1	$ 6,000			
	C	1	$ 4,000	$ 72,000	$ 118,000	$ 190,000

The cheapest way is to realize the project in 14 month in the cost of $ 182,000. If we need to finish the project within 12 months, the cost will be $ 190,000.

Remark

Therefore, to get the duration of the project from 13 to 12, it is necessary to crash two activities. If the project takes 13 months, there are two critical paths, so we need to crash both of them and it is not possible (or too expensive) to do it by crashing only one activity.

LO in Project Management

First, let us choose the variables. Put y_i for the time when the activity i will be finish and y for the time when the whole project will finish. The other variables are the crashing ones – set x_i for the reduction in the time of the activity i due to the crashing of this activity.
Let us assume that we are interested in the cheapest possible way how to complete the project. Then we can formulate the problem:
The objective function presents the total cost. The total cost is the sum of direct costs in normal time for all activities, crashing costs, and indirect costs per project (depending on the duration of the project)

$$\min 5000x_A + 5000x_B + 4000x_c + 6000x_D + 6000y + 88000,$$

subject to

$$\begin{aligned} y_A &\geq 8 - x_A,\ y_B \geq 9 - x_B \\ y_C &\geq y_A + 6 - x_C,\ y_D \geq y_B + 7 - x_D \\ y &\geq y_C,\ y \geq y_D \\ x_A &\leq 3,\ x_B \leq 2 \\ x_C &\leq 2,\ x_D \leq 3 \\ x_A, x_B, x_C, x_D &\geq 0. \end{aligned}$$