

ŘEŠENÍ

V příkladech 1.-7. vyřešte tyto úkoly:

1, 2. homog., 3. - 7. nehomog.

- určete, zda soustava je homogenní nebo nehomogenní
- zapište soustavu rovnic jako matici
- proved'te Gaussovu eliminaci (matici upravte do trojúhelníkového tvaru)
- určete počet řešení soustavy
- najděte všechna řešení soustavy
- soustavu rovnic graficky interpretujte.

$$h = n = 3$$

$$x - 2y - 3z = 0$$

1. $3x + 2y - z = 0$

$$3y + z = 0$$

$$\begin{pmatrix} 1 & -2 & -3 \\ 3 & 2 & -1 \\ 0 & 3 & 1 \end{pmatrix} \xrightarrow{(-3)} \begin{pmatrix} 1 & -2 & -3 \\ 0 & 8 & 8 \\ 0 & 3 & 1 \end{pmatrix} \xrightarrow{(-3)} \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix} \xrightarrow{(-3)} \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix} \xrightarrow{\text{Make 1 in 3rd row}} \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

TRS 1. drehen

$$2x - y + z = 0$$

$$2. \ x + 2y - 2z = 0$$

$$3x + y - z = 0$$

$$\begin{pmatrix} 1 & 2 & -2 \\ 2 & -1 & 1 \\ 3 & 1 & -1 \end{pmatrix} \xrightarrow{(-3)} \begin{pmatrix} 1 & 2 & -2 \\ 2 & -1 & 1 \\ 0 & -5 & 5 \end{pmatrix} \xrightarrow{(-2)} \begin{pmatrix} 1 & 2 & -2 \\ 0 & -5 & 5 \\ 0 & -5 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} h=2 < 3 = n \\ \text{so free var!} \end{array}$$

$$\begin{array}{l} z=t \\ -5y+t=0 \\ y=t \end{array} \quad \begin{array}{l} x=-2t+2t \\ x=0 \end{array} \quad \underline{[0, t, t]}$$

SVAZEK
1. obzrnek

$$2x - y + z = 0$$

3. $4x - 2y + 2z = 1$

$$x + y + z = 5$$

$$\begin{pmatrix} 1 & 1 & 1 & | & 5 \\ 4 & -2 & 2 & | & 1 \\ 2 & -1 & 1 & | & 0 \end{pmatrix} \xrightarrow{\substack{R_2 - 4R_1 \\ R_3 - 2R_1}} \begin{pmatrix} 1 & 1 & 1 & | & 5 \\ 0 & -6 & -2 & | & -19 \\ 0 & -3 & -1 & | & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 5 \\ 0 & -3 & -1 & | & -10 \\ 0 & -6 & -2 & | & -19 \end{pmatrix} \xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & 1 & 1 & | & 5 \\ 0 & -3 & -1 & | & -10 \\ 0 & 0 & 0 & | & 1 \end{pmatrix} \quad h_3 = 2 \neq h_2 = 3$$

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$$4. \quad \begin{cases} 2x + 3y + z = 15 \\ 7x - y + z = 9 \\ x + 2y + z = 9 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 7 & -1 & 1 & 9 \\ 2 & 3 & 1 & 15 \end{array} \right) \xrightarrow{\substack{(-7) \\ (-2)}} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & -15 & -6 & -54 \\ 0 & -1 & -1 & -3 \end{array} \right) \xrightarrow{(-3)} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & 1 & 1 & 3 \\ 0 & -5 & -2 & -18 \end{array} \right) \xrightarrow{(-5)} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & -3 \end{array} \right)$$

TRS
1. *drogula*

$$x + y + 2z = 4$$

5. $x - 2y + z = 0$

$$x - 5y = -4$$

$$\begin{pmatrix} 4 & 1 & 2 & | & 4 \\ 1 & -2 & 1 & | & 0 \\ 1 & -5 & 0 & | & -4 \end{pmatrix} \xrightarrow{(-1)} \begin{pmatrix} 1 & 1 & 2 & | & 4 \\ 0 & -3 & -1 & | & -4 \\ 0 & -6 & -2 & | & -8 \end{pmatrix} \xrightarrow{(-2)} \begin{pmatrix} 1 & 1 & 2 & | & 4 \\ 0 & -3 & -1 & | & -4 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \quad \begin{aligned} h_s &= h_u = 243 = n, \quad z = t \\ x &= 4 - y - 2t \\ y &= 4 - t \\ x &= 4 - \frac{4-t}{3} - 2t \\ x &= \frac{12-4+3t}{3} \\ x &= \frac{8-5t}{3} \end{aligned}$$

SVAZEK
1. dubna

$$2x - 3y + 4z = 8$$

6. $3x + 5y - z = 10$

$$7x - y + 7z = 15$$

$$\begin{pmatrix} 2 & -3 & 4 & | & 8 \\ 3 & 5 & -1 & | & 10 \\ 7 & -1 & 7 & | & 15 \end{pmatrix} \xrightarrow{\substack{(1) \leftrightarrow (2) \\ R_2 - R_1 \\ R_3 - R_1}} \begin{pmatrix} 3 & 5 & -1 & | & 10 \\ 2 & -3 & 4 & | & 8 \\ 0 & 19 & -14 & | & -26 \end{pmatrix} \sim \begin{pmatrix} 2 & -3 & 4 & | & 8 \\ 0 & 19 & -14 & | & -26 \\ 0 & 0 & 0 & | & 1 \end{pmatrix}$$

TRG

$$x + y + z - 1 = 0$$

7. $2x + 2y + 2z = 1$

$$-3x - 3y - 3z + 1 = 0$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 \\ -3 & -3 & -3 & -1 \end{array} \right) \xrightarrow{\substack{R_2 - R_1 \\ R_3 + R_1}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{array} \right) \quad h_5 = 1 \neq h_2 \quad \underline{\text{Ans.}}$$

SVAZEK 2. DOZUKY

8. Určete hodnotu matice:

Uřete hodnotu matice:

$$\begin{pmatrix} 2 & 1 & 3 & -1 \\ 3 & -1 & 2 & 0 \\ 1 & 3 & 4 & -2 \\ 4 & -3 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 4 & -2 \\ 3 & -1 & 2 & 0 \\ 2 & 1 & 3 & -1 \\ 4 & -3 & 1 & 1 \end{pmatrix} \xrightarrow{\substack{(-4) \\ (-2) \\ (-3)}} \begin{pmatrix} 1 & 3 & 4 & -2 \\ 0 & -10 & -10 & 6 \\ 0 & -5 & -5 & 3 \\ 0 & -15 & -15 & 9 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} 1 & 3 & 4 & -2 \\ 0 & -5 & -5 & 3 \\ 0 & -5 & -5 & 3 \\ 0 & -5 & -5 & 3 \end{pmatrix} \xrightarrow{3} \begin{pmatrix} 1 & 3 & 4 & -2 \\ 0 & -5 & -5 & 3 \\ 0 & -5 & -5 & 3 \\ 0 & -5 & -5 & 3 \end{pmatrix}$$

$h=2$

$$h=2$$